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Planck Length, Soriticality, and Zeno's Paradox

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Abstract

Many physicists claim that the Planck length, ℓ_p, is the smallest unit possible. Anything smaller than that would see time and space as constituents of reality breaking down. Does ℓ_p block the infinite series of steps allegedly required for a supertask? The question we will address is: "If it does, how?" A full explication of the logical mechanics of the paradoxes of motion, utilizing a soritically reductive approach, seems to be lacking in the current literature. Taken in isolation, recursive soriticality is the slipperiness of the slippery slope fallacy. A keen eye can see its use not even in lay political polemics, but also in contemporary physics. Planck not only solves an ancient problem, but also inadvertently draws our attention to the wisdom of older forms of thought.

Keywords: paradox of the arrow, paradoxes of motion, Planck length, sorites, supertask, Zeno

I. The Problem

Empirical methods have been some of the most impeccable humanity has had to offer on their journey since the cave. The method of inquiry known as science has shown itself to be a mighty force since its overt endorsement by the enlightenment-era philosopher Sir Francis Bacon in his 1620 *Novum Organum*. The rationalists, with their importance of reason above all else, are also noteworthy. Their stress on a-priori reasoning was beneficial in promoting deductive methods of thought. People like Planck have, as the West's educated elite, excelled in both modes of thinking, and I echo others in that science should be thought of as providing philosophy with a formal solution to Zeno's paradox. Michael Ruse writes:

> It is generally agreed that science is the product of roughly two kinds of reasoning: deductive and inductive (Salmon, 1973; Hempel, 1966). Beginning with deduction, where the scientist is constrained and guided by inference of a formal kind, conclusions follow necessarily from premises. An important branch of such formal reasoning is (deductive) logic, where the scientist is bound, or (if you like) qua scientist agrees to be bound, by certain basic logical principles, together with fixed laws or rules of inference [. . .] Logic alone is not enough for the scientist. His/her concern is with the world around us, the very essence of the scientific system is to bring together empirical claims about this world. But logic constrains and informs the scientist's work. (Ruse, 1998, pp.157-158)

We could go on about this, although the main thrust here is to accept that science's methods are as adept at solving problems as philosophy's. The Planck solution should carry a high level of legitimacy. Many types of solutions to the Greek paradoxes have been tried. The flight of an arrow hitting a target is a supertask, requiring for its completion an infinite number of steps.

The Paradoxes of Motion allow for a seemingly infinite number of recursive steps, although common sense tells us the arrow can cross these distances. Generally, the four solutions we will explore do not capture the soritical dimension of the problem, attribute originally to Eubilides.

Soriticality directly applies to reiterative operations such as those present in the paradoxes of motion. ℓ_p blocks the assumed infinite regress. The transfinite solution is just one out of four which we will explain. How does the arrow exhaust all the steps? The Paradox of the Arrow was only one out of a number of such reductio arguments, known together as the “Paradoxes of Motion”. Zeno of Elea, 5th B.C.E, was following in the footsteps of the then leader of the Eleatic school of philosophy, Parmenides (515-450 BCE), in promulgating the idea that reality is permanent. Change was merely an illusion caused by our senses, for the Eleatics. Ultimately on their view, time and motion cannot exist. Our commonsense notions here are contradictory. (Palmer, 2021) Zeno is mentioned in Plato’s *Parmenides* (515 BCE), and the paradoxes are explored in Aristotle’s *Physics*, 6.9 (400 BCE). Recursive divisional operations are required for a supertask to be completed. Zeno hence persuades his interlocutor that motion is impossible. The Paradox of the Arrow is illustrated below:

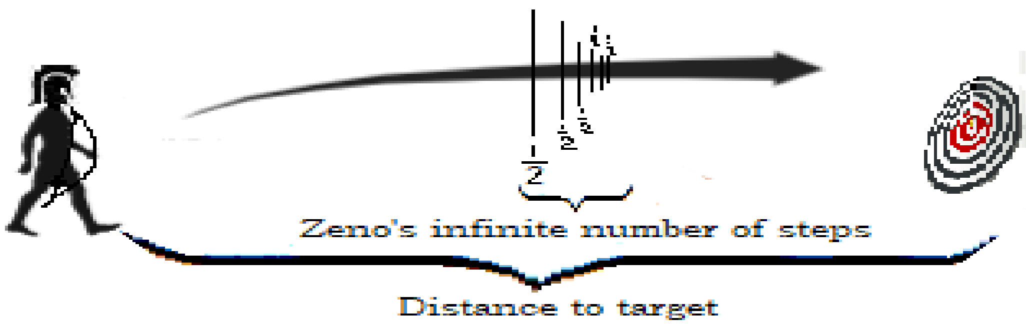

Fig.1 In order for the arrow to hit its target, it must travel half the distance. In order for it to travel half the distance, it must travel half of that, and so on.

II. Four Answers

A number of solutions to this have been attempted (down to an ordinary language analysis of the word “heap”), although my contention is that it is the slippery slope fallacy on its head. There is a contention that a series will not stop, inherent in paradoxes of motion. According to the mathematical solution, if we can antecedently assign the velocity of the arrow an “indeterminate form”, then there is no way we calculate when it will hit the target. “The speed evaluates to v = 0/0, which is an indeterminate form, at any instant.” (Grimbaldi, 1996, p.300) If this is to be taken as the value for the velocity of the arrow, Zeno can make any number of claims. This might seem like a compelling argument although the soritical nature of Zeno’s arguments are not explained with the mathematical approach.

Aristotle claimed that the arrow is in actual rather than potential motion, which was supposed to overcome Zeno’s argument. Russell later argued counter to Aristotle that all there is to motion is being at different places at different times; an object does not move from one place to another, but it is at different places at different times. (Pemberton, 2022) Both appear to capitalize on Zeno’s assumption that antecedently the arrow is supposed to stand still. A final solution that makes its way into the literature invokes the supertask. Achilles may have an infinite of steps to take, although they are not *absolutely* infinite. The distance Achilles has before he reaches the tortoise is *transfinite*.

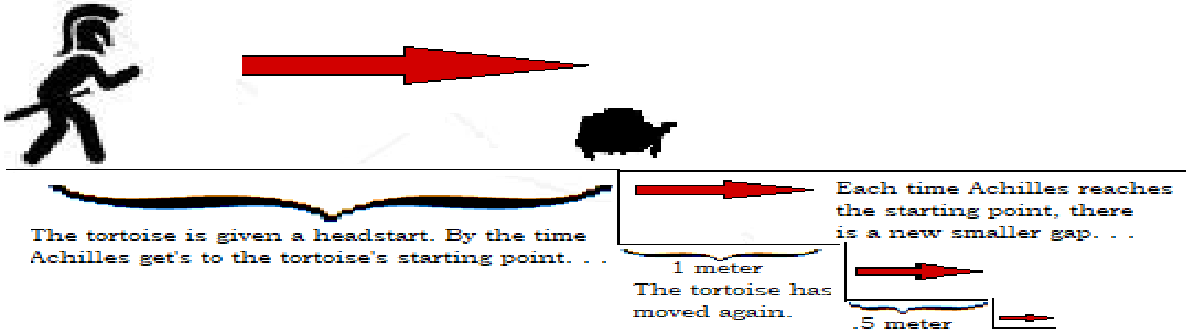

Fig. 2 On Zeno's account, Achilles can never catch up to the tortoise due to the never-ending requisite number of steps. The paradox of the heap illustrates problems with recursive operations which are ongoing as well.

The four approaches to the paradoxes of motion all have their merits. Although none of them have upheld the notion that a series may stop. Reiterative operations must continue in the Paradox of the Arrow and Achilles and the Tortoise. We are not given a reason why the operations shan't cease, although this leaves us in a good position to explore the mechanics of the argument.

III. Soriticality

The slippery slope fallacy is tacitly smuggled into Zeno's arguments. The sorites (*soros* for "heap") was first formulated by Eubulides of Miletus (4th century BCE) in his paradox of the heap. According to it, no matter how many grains there are in an arrangement, they do not constitute a heap. Here is how it works: since "heap" does not apply to N, nor to $N+1$, nor $N+2$, nor $N+3$, then four grains of sand or more will not either, if we let the argument run. We can possibly calculate that N_∞ grains do not constitute a heap. Due to the nature of the series, we conclude that N_n grains will never compose a heap.

The solution of four grains of sand has been offered as a solution, so that the grains may stack, as this is the smallest configuration possible. Aside from that, the important concept involved would be the slippery slope. As Frege mentions, one can see this most easily in respect to "concepts with fuzzy boundaries". (Hyde, 2208, 15) A fallacy in one direction is a fallacy conversely. Tautologically, if ~A, then the assertion *that* A would also be a negation. The mathematical concept of inversion may also be applied to philosophical concepts. Soriticality is the slippery slope fallacy, although it is inverted.

So, for every recursive operation, such as stacking grains of sand or catching up to the tortoise or hitting the target, if for $N_n \neq H$ then $N_\infty \neq$ H. If the first three grains of sand do not make a heap, then no amount will constitute a heap. If Achilles must move forward 1 meter, he must move half of that, and so on recursively for that operation (see fig.2).

IV. The Plank Length

Recursive soriticality must stop at some point if there is to be a bona fide solution to the Paradoxes of Motion. In the introduction, we claimed ℓ_p, blocked infinite regression in these cases. How does it do this? There are reasons why ℓ_p is the smallest length possible. At any measurements smaller than the Planck length time and space cease to exist. There cannot be anything between two individual Planck lengths, and it is impossible to determine a distance between a Planck length apart. "According to modern atomists, the Planck length is the diameter of an indivisible particle" (Haugh, 2018) We can see why a meter is a meter and how Achilles can cross this as well, if we think in utterly realist terms given Max Plank's discovery.

There is a point at which the divisions will stop if we accept ℓ_p. Physicists claim this is the smallest unit possible at 1.616 255(18) × 10−35 m. The argument that the arrow must traverse over an infinity of tiny subdivisions is thwarted. The transfinite analysis is a shorthand for a disbelief in the idea of a scientific measure of a nonrelative or nontrivial length. The Plank length, d=1 would be the length the arrow must travel that cannot be halved any further. It is 100 quintillionths of a proton's diameter. It might be the case that in order for this to be a real issue, both Achilles and the tortoise might have to shrink down to this size as well.

Conclusion

Our soritical analysis persuaded us to see that the previous ways of addressing Zeno were reliant on techniques of sidestepping the problem, rather than solving it. If the current state of literature is any indication, philosophers have broken away from the soritical analysis requisite in seeing the mechanism by which the paradoxes of motion get their bite, i.e., recursive soriticality. After seeing this, the appropriate question is, then, “when”? Is the regress finite? Max Planck’s introduction of the smallest measurement possible answers the *when*. We have considered some legitimate ways of addressing the paradoxes of motion, and not one stressed that soritical recursiveness is not by any means a necessary series of infinite operations. To think this way would be to think fallaciously. It can be scientifically proven with the Planck length.

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